On the characterizations of cofinite complexes
Appendix

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The four questions are proposed in the paper [H1, §2] over regular rings. Especially, the following was given as the fourth question:

**Question** (Hartshorne 1970)

(Fourth Question) Let $R$ be a regular ring of finite Krull dimension and $I$ an ideal of $R$. Suppose that $R$ is complete with respect to an $I$-adic topology. Then does there exist an abelian subcategory $\mathcal{M}_{cof}$ consisting of $R$-modules, such that $I$-cofinite complexes $N^\bullet$ are characterized by the property “$H^i(N^\bullet) \in \mathcal{M}_{cof}$” for all $i$?
It is well known that there is a counter example for the fourth question (cf. [H1, § 3, An example, p. 149]).

**Example (Hartshorne)**

Let ring $R$ and the ideal $I$ be:

$$ R = k[x, y][[u, v]], \quad \text{and} \quad I = (u, v). $$

Then the answer of the fourth question is negative.

The ring $R$ is the formal power series ring over the polynomial ring $k[x, y]$. Notice that the ideal $I$ is of dimension two and generated by two elements in $R$. 
Our theorems propose the positive answer to the fourth question, provided that the ideals are of dimension one or principal over a homomorphic image of a (not necessarily local) Gorenstein ring $A$ of finite Krull dimension, where $A$ is complete with respect to the $J$-adic topology.


